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Numerical modelling of critical conditions for the onset of a limit state in the rock mass surrounding unfilled underground voids in iron ore deposits

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Abstract. The purpose of this study is to develop an approach for the quantitative assessment and prediction of rock mass condition surrounding unfilled underground voids, with a focus on a limit state associated with potential instability and ground surface collapse, using numerical modelling techniques. The investigation was carried out using finite element modelling of the stress-strain state of a stratified rock mass in the RS2 software package. To adequately reproduce the mechanical behaviour of fractured rocks of the Kryvyi Rih Iron Ore Basin, the stratified geological structure of the rock mass was incorporated into the model. The nonlinear Hoek-Brown failure criterion was applied, accounting for the Geological Strength Index (GSI). The existence of a transitional (near-failure) geomechanical state of the rock mass surrounding unfilled underground voids has been established. This state develops between stable and unstable conditions and is characterised by mechanical interaction between the void and the ground surface. A stable logarithmic relationship between the lower and upper bounds of the critical ratio H/L_c and the void depth has been identified, quantitatively reflecting the increase in rock mass resistance as the depth increases toward the limit state. An exponential relationship between the width of the ground surface subsidence trough and the parameter H/Lc has been identified, enabling the prediction of the extent of the potential collapse zone. An exponential relationship between the required strength of the cemented paste backfill and the H/L_c ratio has been established, defining the minimum bearing capacity of the backfill under near-failure conditions. For the first time, the existence of a distinct near-failure geomechanical regime of the rock mass surrounding unfilled underground voids has been quantitatively substantiated as an independent state preceding progressive ground surface collapse. The obtained relationships enable predicting the geomechanical condition of the rock mass above unfilled voids, determining the range of their critical geometric parameters, and timely identifying voids in a near-failure state. The developed approach can be applied in engineering practice to justify the parameters of cemented paste backfill from the surface to prevent sudden ground surface collapse.

Keywords: *numerical modelling; underground void; stratified heterogeneous rock mass; Hoek-Brown failure criterion; near-failure state; ground surface collapse; cemented paste backfill.*

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1. Introduction

Ukraine possesses strategically important proven iron ore reserves. More than 45% of these reserves (8.6 billion t) are located in Dnipropetrovsk Oblast, with the greater part concentrated within the large Kryvyi Rih Iron Ore Basin (Kryvbas). The mining and processing of iron ores form the backbone of Ukraine's metallurgical industry. In contrast, the produced iron ore feedstock and concentrate are supplied to both the domestic market and export. A substantial share of the extracted ore and beneficiation products is exported, generating significant foreign-currency revenues for the state budget and supporting the country's external economic balance; in 2021, these revenues amounted to USD 3.91 billion. At present, approximately 70% of Ukraine's total iron ore production is mined annually in the Kryvyi Rih Iron Ore Basin, with 90% extracted by open-pit mining and 10% by underground mining [1-3].

Over more than a century of intensive iron ore mining in the Kryvyi Rih Basin, the subsurface and the ground surface have undergone large-scale anthropogenic transformation. In the rock mass, a complex set of geomechanical processes has developed, including the formation of extensive zones of strata movement and discontinuities, the accumulation of unfilled underground voids, and a progressive deterioration of rock mass stability. At the surface, gradual soil depletion and degradation, destruction of natural ecosystems, and large-scale stockpiling of mining and beneficiation wastes are observed, accompanied by contamination of groundwater, surface waters, and the atmosphere [4-8]. Under these conditions, the design and implementation of environmentally oriented mining technologies become important tasks [9-12].

The most pressing issue remains elevated anthropogenic and geomechanical hazards, manifested by recurrent sudden rockfall events and slope failures, the formation of collapse zones and subsidence troughs, and the resulting threats to infrastructure, the environment, and public safety. Owing to the long-term operation of deposits in the Kryvyi Rih Iron Ore Basin, many of which were developed by underground mining without adequate closure of the mined-out space, a considerable number of blind and isolated ore bodies and lenses have been extracted, and the resulting voids pose a significant geomechanical and anthropogenic risk to the region.

At present, approaches to assessing the stability of a rock mass containing voids created by the extraction of ore deposits, as prescribed in existing scientific and methodological documents [13, 14], are generally based on determining the conditions for the formation of primary sinkholes. These approaches allow only the identification of the transition of the system into an unstable state, with a likelihood of deformation reaching the ground surface in the form of collapse funnels, and effectively reduce the geomechanical state of the rock mass to two categories: stable and unstable.

However, as practical experience in the Kryvbas demonstrates, sudden ground surface collapses are periodically recorded even after the extraction of reserves from blind, isolated ore bodies has been completed [15, 16]. The occurrence of such collapses indicates the presence of an intermediate state, namely, a near-failure state of the rock mass surrounding unfilled underground voids, in which the rock mass is weakened but still retains residual stability and is capable of an abrupt transition to an unstable regime. These formed and unfilled underground voids may suddenly destabilise, leading to sinkholes or the development of subsidence troughs at the ground surface. These sudden events are particularly hazardous in urbanised areas, where industrial or civil infrastructure, as well as natural assets, may be located above the voids. Therefore, there is a need for geomechanical forecasting that enables the identification of underground voids in a near-failure state and supports scientifically grounded decisions on their timely stabilisation.

The importance of timely identification of formed voids that are in a near-failure state lies in the opportunity to implement preventive measures and thereby avert ground surface collapse. A practical mitigation measure is the placement of cemented paste backfill into unfilled voids from the ground surface through a drilled backfilling borehole [4, 17-19], as such voids are typically located at a relatively shallow depth. The paste backfill mixture is prepared from fine-grained iron ore beneficiation tailings (65-80%) recovered from dewatered tailings storage facilities, locally available binders (3-10%) such as cement, ground slag, and fly ash and 15-30% water [20]. These components are mixed at a surface backfilling plant and then delivered by pressurized pipeline transport to technogenic voids, in this case, unfilled underground voids left after the extraction of blind, separated, and isolated ore bodies.

Identification of a near-failure rock mass state surrounding unfilled voids is advisable using finite-element numerical modelling in modern geomechanical software packages. Studies by Kryvyi Rih researchers have made a significant contribution to understanding the deformation patterns of the rock mass during ore extraction [21-23]. However, most investigations were local in scope, focused on stoping chambers, and were performed mainly under the assumption of linear-elastic rock mass behaviour. Given the hard-rock and fractured nature of the Kryvbas formations, the application of generalised nonlinear strength criteria, particularly the Hoek-Brown failure criterion, is promising. This approach provides a more accurate representation of the influence of rock mass structural disturbance and improves the prediction of strata movement zones and potential collapses.

2. Materials and methods

At present, the following research approaches can be distinguished for solving geomechanical problems in mining engineering: analytical methods [24, 25], physical modelling [26-28] and numerical modelling [29-32]. The most widespread approach is finite element numerical modelling due to its ability to realistically reproduce phenomena and processes, provide reliable results, and reduce labour intensity.

One of the most suitable strength criteria for hard rock and fractured rock masses is the empirical Hoek-Brown failure criterion, which accounts for rock mass quality through the Geological Strength Index (GSI), jointing, and the degree of disturbance. E. Hoek and E. T. Brown developed the Hoek-Brown criterion in 1980 [33]. It was proposed as an empirical alternative to the Mohr-Coulomb criterion to describe the strength of intact rock and rock masses exhibiting nonlinear behaviour, while accounting for fracturing, structure, and disturbance.

Today, the Hoek-Brown failure criterion is widely used for modelling rock mass behaviour in geomechanical analyses. Its direct implementation is available in major specialised software packages, including FLAC3D, RS2, RS3, and Slide2/3 (Rocscience), as well as UDEC and 3DEC (Itasca), where it is provided as a core function with options to account for strength degradation and staged excavation.

Within this study, the numerical calculations were performed using the RS2 software package, which provides a range of strength criteria, including the Hoek-Brown criterion [34]. A key parameter of the Hoek-Brown criterion is the Geological Strength Index (GSI), which provides an integrated assessment of rock mass structural disturbance and accounts for scale effects.

To model the stability of rocks surrounding unfilled voids in the Kryvbas, the GSI values for different lithologies were estimated as GSI = 3045 based on an analysis of the rock mass structure and the condition of discontinuity surfaces observed in exposures within collapse zones, using photo documentation and supplementary data. This range is justified because it reflects a combined weakening effect relative to the natural rock mass state, for which GSI is typically assessed as 60-70 increased weathering due to the shallow depth of the voids, and additional anthropogenic damage to the rock mass caused by historical blasting during mining operations.

For the study, a geomechanical model was developed that includes unconsolidated overburden deposits, a stratified hard-rock mass, and an unfilled underground void left after the extraction of a blind, isolated ore body, according to the averaged stratigraphic section of the central (Saksahan) iron ore district of the Kryvbas. The model domain was set to 600 m in the vertical direction and 800 m in the horizontal direction.

The unfilled void considered in this work is located within the V iron-bearing horizon, which is characterised by the highest ore-bearing coefficient and the greatest concentration of ore bodies in the Saksahan district of the Kryvbas. Given an average thickness of about 60 m for the rocks of the V iron-bearing horizon and an average ore body thickness of

20-30 m, it is evident that, in a cross-strike section toward the hanging wall, the model must include rock layers of adjacent iron-bearing and shale horizons up to the lateral boundary of the domain; therefore, the geomechanical model is represented as a stratified rock mass.

In the model, the stratigraphic sequence starts with arkosic sandstones of the Skelevatska Suite. Upsection, chlorite-amphibolite schists represent the III-V shale horizon. This is followed by an alternation of productive and barren units: the V iron-bearing horizon consists of martite ores and jaspilites; the VI shale horizon is composed of silicate schists; and the VI iron-bearing horizon comprises martite hornfels. The sequence in the model is completed by the VII shale horizon, which contains chlorite-biotite schists with barren hornfels, and the VII iron-bearing horizon, represented by amphibole-magnetite and martite hornfels. The constructed finite element model is shown in Figure 1.

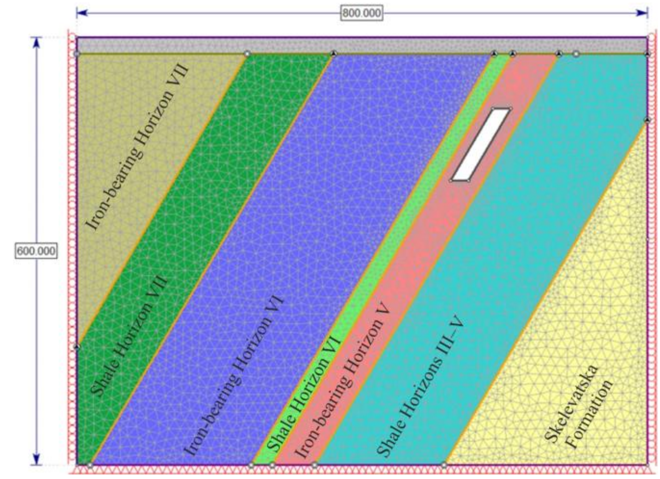

Figure 1. Finite element model of a stratified rock mass containing an unfilled underground void

The numerical modelling scheme involves systematically varying the geometric parameters of underground voids formed by mining blind ore bodies to assess rock mass stability. Five depths of the void roof below the ground surface are considered (50, 100, 150, 200, and 250 m), with an ore body dip of 60° and an average horizontal ore body thickness of 25 m. The void length along the dip is varied as follows: an initial excavation length of 50 m is modelled, after which the length is gradually increased in 10-15 m increments until a size is reached at which the rock mass and the ground surface lose integrity completely.

The geomechanical model employs the physical and mechanical properties of Kryvbas rocks, generalised from the scientific and technical literature. Because the behaviour of the fractured hard-rock mass is described using the Hoek-Brown failure criterion, the built-in RS2 calculator is used to automatically convert the laboratory properties of intact rock (uniaxial compressive strength, σ_{ci}, and Young's modulus, E_i) and rock mass quality indices (GSI, D) into the dimensionless Hoek-Brown parameters required for analysis (mb, s, and a). This approach ensures the correct application of the criterion based on the fundamental rock characteristics. For each lithology, the model specifies both the derived rock mass parameters according to the adopted Hoek-Brown failure criterion (computed using the above procedure) and additional general mechanical properties (density, deformation modulus, compressive strength, and Poisson's ratio). The generalised physical and mechanical properties are presented in Table 1.

In addressing the stated geomechanical problem, rock strength is not treated as a variable parameter. This is because, for relatively weak rocks ($f < 5$), the rock mass above unfilled voids created by mining blind ore bodies several decades ago would, with a high probability, have already collapsed. However, where rock strength is higher ($f = 10$-15), a void may remain in either a stable or a near-failure (collapse-prone) state depending on several controlling factors, including the void depth, the horizontal thickness of the extracted ore body, its extent along the dip, and the dip angle. Therefore, numerical modelling with systematic variation of these conditions enables the identification of potential stress-strain states of the rock mass and the assessment of the risk of ground surface collapse.

Within this study, a two-factor criterion-based approach is proposed to determine the critical void length along the dip, L_c. This approach allows defining the range corresponding to a controlled near-failure state of the rock mass. The bounds of this range are determined by:

– the onset of an unacceptable risk to surface facilities due to ground surface damage, U_{surf};

– the loss of geomechanical stability of the void roof (crown contour), U_{cont}. Together, these criteria define an interval of critical void lengths along the dip within which preventive measures, timely stabilisation/closure of the void using a backfill mixture, are required [35].

Table 1. Physical and mechanical properties of rocks based on the modified Hoek-Brown failure criterion

Rock type	Suite, horizon	GSI	m_b	S	a	E_i, GPa	γ, t/m^3	σ_{ci}, MPa	E_i, GPa	ν
Arkosic sandstones	Skelevatska suite	45	1.99 1.16	0.0012 0.00024	0.51 0.53	7.8	2.63	130	49	0.25
Chlorite-amphibolite schists	III-V shale horizon	30	0.49 0.29	0.00041 0.000079	0.52 0.56	3.0	3.0	110	36	0.22
Martite ores	V iron-bearing horizon	45	1.68 0.98	0.0022 0.00042	0.5 0.52	7.4	3.8	65	33	0.2
Martite jaspilites		45	2.1 1.23	0.0022 0.00042	0.5 0.52	21.2	3.45	170	95	0.18
Silicate schists	VI shale horizon	40	0.7 0.41	0.0012 0.00024	0.51 0.53	7.9	3.2	100	50	0.21
Martite hornfels	VI iron-bearing horizon	35	1.86 1.09	0.00073 0.00014	0.51 0.54	9.1	3.36	150	80	0.21
Chlorite-biotite schists with barren hornfels	VII shale horizon	38	1.09 0.64	0.001 0.00019	0.51 0.53	5.5	2.9	85	39	0.24
Amphibole-magnetite and martite hornfels	VII iron-bearing horizon	40	1.99 1.16	0.0012 0.00024	0.51 0.53	15.1	3.25	140	95	0.21

The first criterion is defined as $U_{surf} \geq 0.2$ m and is based on principles of structural mechanics and safety, and it correlates with regulatory limit deformations for subgrades and foundations [36-38]. Exceeding this threshold indicates the onset of unacceptable deformations that pose a direct risk to the serviceability and integrity of industrial buildings and structures, and also lead to degradation of valuable soils.

The second criterion is the limit displacement of the void contour, U_{cont}, which represents the loss of structural integrity of the void roof (crown) associated with instability of the exposed hanging-wall rocks. The limit displacement is taken as 15% of the horizontal ore-body thickness ($m_h = 25$ m). In our case, $U_{cont} \leq 3.75$ m [39-41]. The adopted displacement limit separates two fundamentally different geomechanical states of the rock mass: before reaching this limit, the roof retains residual load-bearing capacity and behaves as a single quasi-stable structure; once the limit is exceeded, local shear and fracture zones coalesce into a continuous slip surface, and the roof enters a stage of progressive, uncontrolled failure, with the likelihood of ground surface collapse depending on the void depth [42-44].

Geomechanical assessment of the rock mass condition is performed using three indicators that enable both quantitative and qualitative characterisation of the stability of the "void-ground surface" system:

– SF_{surf} is the ground-surface strength factor above the void, which reflects the ratio of the available shear strength of the rocks to the actual shear stresses acting in the near-surface layer of the rock mass;

– U_{surf} is the maximum vertical settlement of the ground surface, indicating the degree of its deformation;

– U_{cont} is the maximum displacement of the contour of the hanging-wall rocks as a response to the loss of support due to the formation of the void.

To justify the required strength of the cemented paste backfill for closing an unfilled void, the contours of the maximum and minimum principal stresses are analysed directly along the rock mass contour, and their quantitative values are obtained using the query function.

3. Results and discussion

As a result of the numerical calculations, a large set of total displacement contours and strength factor contours was obtained for void roof depths in the range of 50-250 m (at 50 m intervals). To avoid excessive graphical loading, contour plots are presented only for the case in which the void roof is located at a depth of $H = 50$ m below the ground surface. For greater depths, the main geomechanical trends are discussed in the text. An analysis of the numerical modelling results for an ore body dip angle of $\alpha = 60°$ and a depth of $H = 50$ m reveals the following features (Figure 2).

To accurately determine the void length along the dip that characterises the near-failure state range and satisfies the adopted criteria ($U_{surf} \geq 0.2$ m, $U_{cont} \leq 3.75$ m), linear interpolation between discrete modelling steps was applied. Because the numerical analysis was performed using fixed increments of the void length along the dip ($L = 52.1$; 78.1; 91.2; 104.2 and 117.2 m), this approach made it possible to reliably estimate the critical length L_c within the interval between adjacent iterations. As a result, the onset of stability loss in the system could be identified with improved mathematical precision, and the lower and upper bounds of the near-failure state could be established.

At a depth of $H = 50$ m, the rock mass exhibits a progressive instability mechanism. Up to a void length of $L = 78.1$ m, the system remains in a stable state (SF > 2.7, $U_{surf} \approx 0$). However, at $L = 91.2$ m, the strength factor decreases to SF = 1.48, ground surface settlement appears ($U_{surf} = 0.26$ m), and local contour displacement develops ($U_{cont} = 1.01$ m). This indicates the initiation of a deformation path within the hanging wall.

Further extension of the void to $L = 104.2$ m leads to a pronounced increase in deformations ($U_{surf} = 2.75$ m, $U_{cont} = 6.0$ m), while at $L = 117.2$ m, a limit state is reached with rock mass collapse ($U_{surf} = 6.9$ m, $U_{cont} = 14.5$ m, SF = 1.0). An increase to $L = 130.2$ m results in complete failure: U_{surf} increases to 12.0 m, and contour displacements reach 24.0 m.

At a depth of $H = 100$ m, the failure stages shift to larger void lengths. Specifically, at $L = 104.2$ m, the system remains stable (SF > 2.6), whereas at L = 117.2 m, a deformation path is already formed ($U_{surf} = 1.2$ m, $U_{cont} = 12.5$ m). The critical state is reached at $L = 130.2$ m (SF = 1.0, $U_{surf} = 3.75$ m, $U_{cont} = 12.6$ m).

At $H = 150$ m, the rock mass shows a further increase in resistance to deformation: the limit state is recorded only at $L = 143.3$ m, while ground surface settlement remains moderate ($U_{surf} = 6.7$ m) relative to internal displacements ($U_{cont} = 24.6$ m), indicating the formation of a natural arch (self-supporting roof).

At depths of $H = 200$-250 m, the system transitions to an internal collapse regime without breakthrough to the ground surface. Even at the maximum analysed void length ($L = 169.3$ m), substantial internal deformations are recorded ($U_{cont} = 26$-28 m), whereas the ground surface remains almost unchanged ($U_{surf} = 1.1$-1.2 m). This indicates a fully developed arching effect and effective attenuation of deformation transmission to the surface.

It is important to note that the loss-of-stability process becomes more abrupt: the system remains stable over a relatively long interval, after which even a minor further increase in void length triggers an avalanche-like transition to complete rock mass collapse, potentially accompanied by the propagation of deformations to the ground surface.

At an ore body (and bedding) dip angle of 60°, the failure mechanism is combined, in which high shear stresses are superimposed on bending stresses. Failure initiates at the void contour, with tensile cracking and the onset of hanging-wall sliding. As a result, the collapse channel propagates along an inclined trajectory, and an asymmetric subsidence trough forms at the ground surface.

Based on the step-by-step analysis of the numerical modelling results, the transition point of the system from a stable state to an intensive deformation-development phase was identified. Ranges of the critical void length along the dip, L_c, were determined, quantitatively characterising the attainment of a near-failure rock mass state. These ranges are summarised in Table 2.

Because the near-failure rock mass state is reached predominantly within the void-depth interval of $H = 50$-150 m, additional simulations were performed at depths of 75 m and 125 m to provide a more detailed representation of deformation development in the rock mass between the void and the ground surface. This densified depth increment yielded more reliable data. The results for the variation of the void length along the dip at $H = 75$ m and $H = 125$ m complement those obtained for $H = 50$, 100, and 150 m.

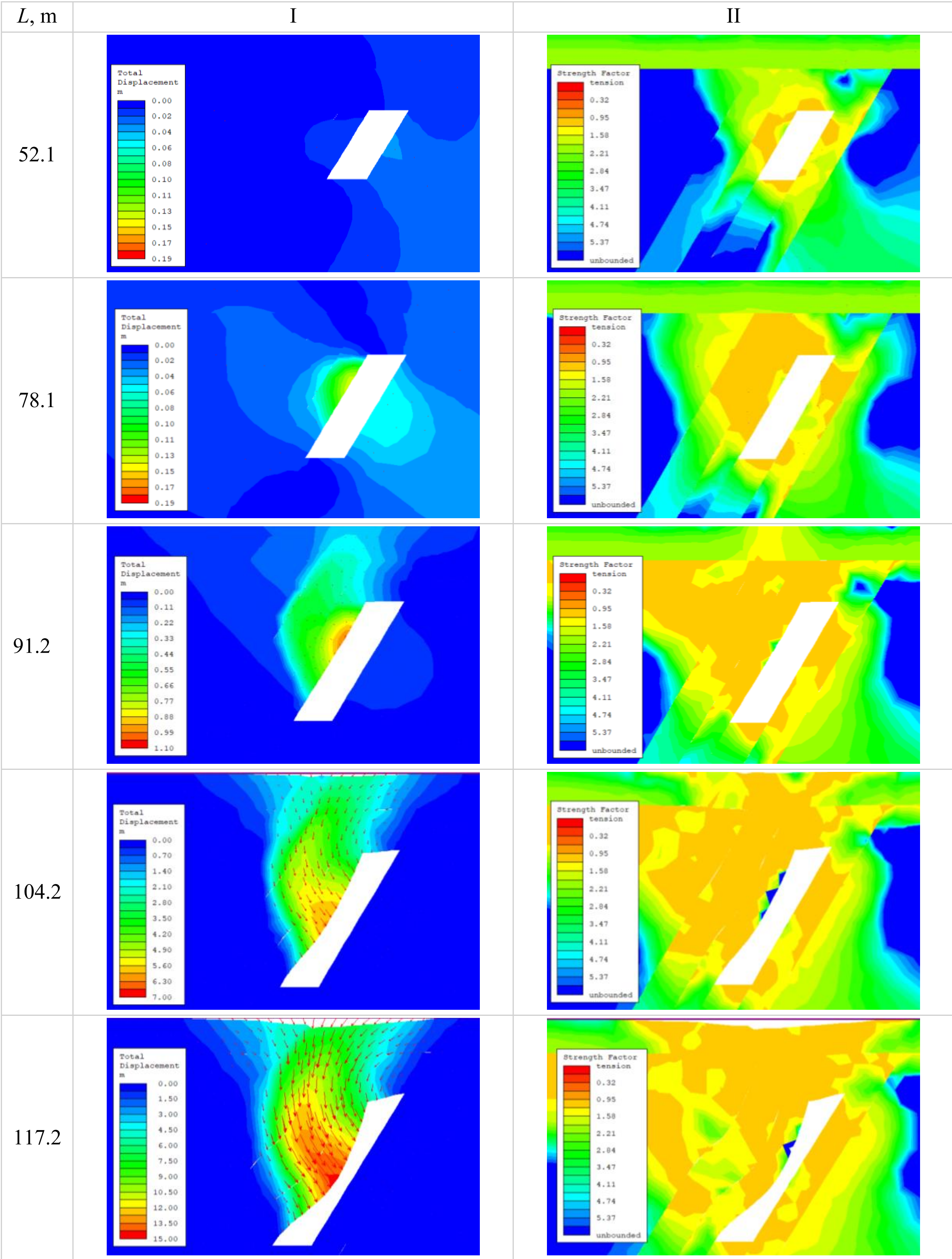

Figure 2. Numerical modelling results (α = 60°): I – total displacement contours; II – strength factor contours

Table 2. Determined ranges of the critical void length along the dip, Lc, characterising the near-failure state of the rock mass between the void and the ground surface

Dip angle, °	Depth (H), m	Range			
		Lower bound, m		Upper bound, m	
		L_c	H/L_c	L_c	H/L_c
60	50	96.2	0.52	106.4	0.47
	75	107.1	0.7	113.6	0.66
	100	117.6	0.85	120.5	0.83
	115	122.3	0.94	125.0	0.92

The conducted set of studies makes it possible to identify regularities in the evolution of the geomechanical state of the rock mass from the formed unfilled underground void (Figure 3), created by mining blind and isolated ore bodies, up to the ground surface, and to directly determine the near-failure state of the rock mass under conditions where sudden ground surface collapse is likely to occur, thereby supporting more reliable prediction and timely implementation of preventive stabilisation measures.

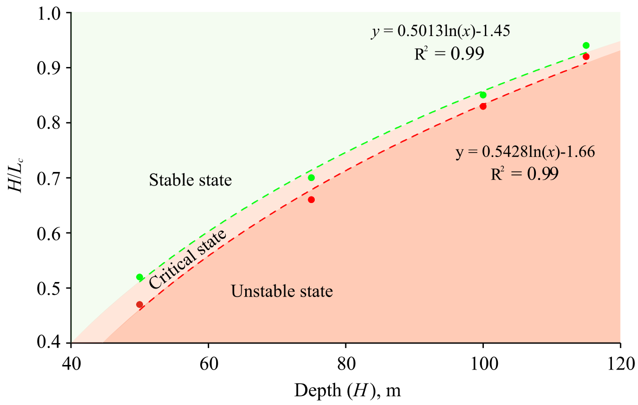

Figure 3. Relationships between the lower and upper bounds of the critical ratio H/Lc and the void depth H for an ore body dip angle of 60°

Analysis of Figure 3 shows a stable nonlinear relationship between the void depth H and the stability parameter H/L_c. The high coefficients of determination ($R^2 = 0.99$) indicate that logarithmic functions accurately approximate these relationships. The main identified regularity is that, as H increases, the ratio H/L_c also increases, directly indicating an increase in geomechanical stability with depth: transition to a limit state at greater depth requires a substantially larger critical void length L_c. Thus, for a dip angle of 60°, the bounds of H/L_c increase from 0.47-0.52 at $H = 50$ m to 0.92-0.94 at $H = 115$ m.

The numerical modelling results also make it possible to predict the ground surface area that is likely to be affected in the absence of preventive measures such as cemented paste backfilling, as well as the potential transition of a void-containing rock mass from a near-failure state to progressive collapse due to degradation of mechanical properties under external factors. During simulations of the near-failure-to-collapse transition, the subsidence trough width across the strike of the ore body was recorded for each geomechanical model. These values were then correlated with the H/L_c ratio and the ore body dip angle α. To establish the governing trends, the upper bound of the near-failure state closest to the onset of rock collapse was adopted. The resulting relationships are shown in Figure 4.

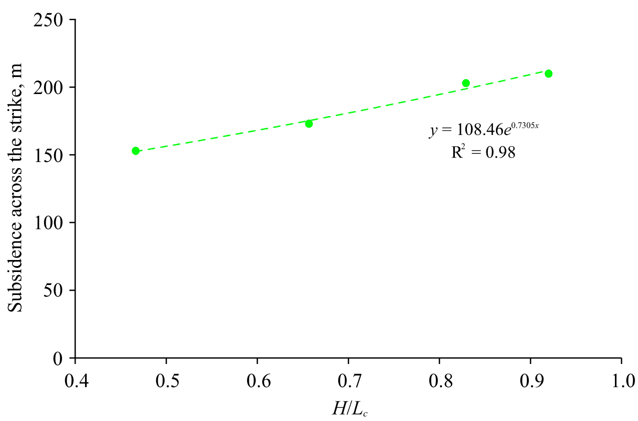

Figure 4. Relationship between the ground surface subsidence trough width and the H/Lc ratio at α = 60° under progressive collapse conditions

The analysis of Figure 4 indicates that the subsidence trough width across the strike of the ore body exhibits a stable exponential increase with increasing geomechanical parameter H/L_c, which is confirmed by the obtained regression equations with high goodness of fit ($R^2 = 0.98$). The area of the potential collapse zone can be estimated by multiplying the across-strike subsidence trough width determined from this relationship by the extent of the underground void along the strike. It was established that during the transition to progressive collapse at a dip angle of 60°, the surface subsidence trough width varies between 150 and 210 m.

To determine the required strength of the backfill mass, a preliminary analysis of the stress distribution after weakening of the surrounding rocks is performed. First, the zone where the load is actually transferred to the backfill is identified, i.e., without the influence of arching effects and local clamping zones near the void corners. In the edge regions of the void, σ_1 may increase artificially due to the contact geometry; however, such stresses do not represent the load that will act on the backfill mass.

The realistic loading on the backfill is associated with the central part of the void contour, away from the corner clamping zone, where $\sigma_3 \approx 0$ and σ_1 are governed primarily by self-weight and by the reduced load-bearing capacity of the hanging wall after degradation. The degraded rock mass zone that will impose the load on the backfill can be delineated by analysing the maximum shear strain contours, in which the constraint region in the corner parts of the void is clearly identifiable. An example is shown in Figure 5.

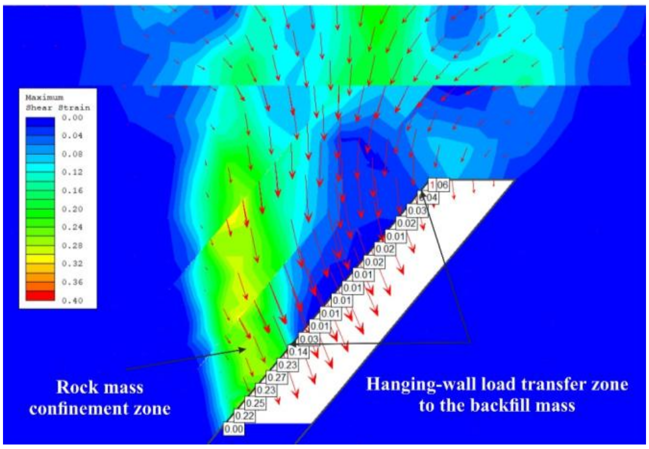

Figure 5. Delineation of the zones of effective rock-to-backfill loading and corner clamping within an underground void based on maximum shear strain contours under rock mass weakening to a collapse state

Analysis of the stress state in the central part of the hanging-wall contour (Figure 5) indicates the development of conditions close to uniaxial compression, where the minimum principal stress $\sigma_3 \approx 0$. Under such conditions, the maximum principal stress σ_1 governs the critical load acting on the backfill mass, which it must sustain without loss of integrity. Therefore, the required uniaxial compressive strength (UCS) of the backfill should satisfy the condition UCS $\geq \sigma_1$.

After identifying the near-failure state of the void for each combination of the H/L_c parameters, the maximum value of σ_1 was determined along the void contour within the effective loading zone (Figure 6), where $\sigma_3 \approx 0$. This value was taken as the required UCS of the backfill mass. Processing the obtained data enabled establishing a relationship between backfill strength and the H/L_c ratio.

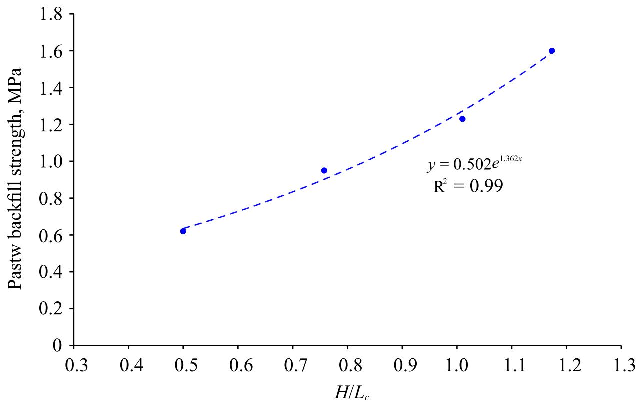

Figure 6. Relationship between the required cemented paste backfill strength and the H/Lc ratio for an ore body dip angle of α = 60°

Analysis of Figure 6 shows a clear exponential relationship between the required cemented paste backfill strength and the geomechanical parameter H/Lc, which characterises the rock mass stress level. As the H/L_c ratio increases (i.e., as the rock mass approaches a near-failure state), the required backfill strength increases nonlinearly, indicating a sharp rise in loads acting on the supporting structure as stability is lost.

Thus, based on the conducted numerical investigations and the derived relationships (Figures 3 and 6), it was established that unfilled underground voids formed by mining blind and isolated ore bodies may be associated not only with stable and unstable geomechanical states of the rock mass, but also with a distinct near-failure state. This state represents the onset of integrity loss and is characterised by a developed deformation coupling between the void and the surface.

The near-failure state occurs within the critical range H/L_c = 0.5-1.2 and is defined by lower and upper bounds that increase logarithmically with void depth. In contrast, the interval between these bounds decreases with increasing ore body dip angle. In this state, the required cemented paste backfill strength exhibits an exponential dependence on H/Lc, increasing as the dip angle decreases. The obtained relationships provide a scientifically grounded basis for identifying voids in a near-failure state, selecting the required backfill strength, and proactively preventing the development of progressive ground surface deformations.

A promising direction for further research is to extend the present analysis to establish more general regularities governing the transition of the rock mass to a near-failure state. In particular, it is advisable to perform similar numerical modelling for ore bodies with other dip angles, such as 50° and 70°, which would allow a quantitative assessment of the influence of this crucial geometric factor. A comprehensive synthesis of the resulting data would support the development of a more universal and reliable methodology for predicting the stability of rock masses above unfilled underground voids and for improving the robustness of geomechanical risk assessment.

For the large iron ore basin considered in this study, a topical problem is the development of deep ground-surface collapse zones induced by active underground mining. Therefore, it is reasonable to numerically model the further evolution of collapse zones and hanging-wall deformations as the mining depth increases, and to evaluate how the formation of a monolithic cemented paste backfill mass within the created voids can alter the geomechanical setting as an engineering measure to block or attenuate deformation propagation.

4. Conclusions

A numerical modelling methodology was developed based on a two-dimensional finite-element geomechanical model of the stratified hard-rock mass of the Kryvbas, which contains an unfilled underground void. The model accounts for the failure behaviour of fractured rocks by applying the Hoek-Brown failure criterion and representing the actual weakened rock mass condition using a justified Geological Strength Index (GSI). This modelling approach enables parametric studies by systematically varying void depth and size, allows identification of the conditions under which the rock mass enters a hazardous near-failure state, and provides direct predictions of ground surface stability.

It was established that unfilled underground voids formed by mining blind and isolated ore bodies are associated not only with stable and unstable geomechanical states of the rock mass, but also with a distinct near-failure state at the threshold of integrity loss, characterised by a developed deformation linkage with the ground surface. The near-failure state develops within the critical range H/L_c = 0.5-1.2 and is bounded by lower and upper limits that increase logarithmically with increasing void depth. In contrast, the interval between these bounds decreases as the ore body dip angle increases. Within the near-failure state, the required cemented paste backfill strength exhibits an exponential dependence on H/L_c, increasing as the dip angle decreases.

Author contributions

Conceptualization: MP; Data curation: KS; Formal analysis: MP; Funding acquisition: MP; Investigation: MP, KS; Methodology: MP; Project administration: MP; Resources: KS; Supervision: MP; Validation: MP, KS; Visualization: KS; Writing – original draft: MP, KS; Writing – review & editing: MP, KS. All authors have read and agreed to the published version of the manuscript.

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Conflicts of interest

The authors declare no conflict of interest.

Data availability statement

The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding author.

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Темір кені кен орындарындағы жерасты қуыстарының айналасында тау жыныстары массивінің шекаралық күйінің қалыптасуының критикалық шарттарын сандық модельдеу

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Аңдатпа. Жұмыстың мақсаты – сандық модельдеу негізінде орнықтылықтың жоғалуына және жер бетінің опырылуына қауіпті шекті күйдегі, жойылмаған жерасты қуыстарының айналасындағы тау жыныстары массивінің жай-күйін сандық бағалау және болжау тәсілін әзірлеу. Зерттеу RS2 бағдарламалық кешенінде соңғы элементтер әдісімен қабатты жыныстар массивінің кернеулі-деформацияланған күйін (КДК) сандық модельдеу арқылы орындалды. Кривой Рог теміркен бассейніндегі жарықшақты жыныстардың механикалық мінез-құлқын барабар қайта жаңғырту үшін массивтің стратиграфиялық ерекшеліктері ескерілді. Геологиялық беріктік индексін (GSI) ескере отырып, Хоек–Браунның сызықтық емес беріктік критерийі қолданылды. Жойылмаған жерасты қуыстарының айналасында орнықты және орнықсыз күйлердің арасында қалыптасатын, жер бетімен деформациялық байланысының болуымен сипатталатын тау жыныстары массивінің шекаралық (бұзылуға жақын) геомеханикалық күйі бар екені анықталды. Қуыстың жату тереңдігіне байланысты H/Lкр критикалық қатынасының төменгі және жоғарғы шекараларының орнықты логарифмдік тәуелділігі анықталды; бұл тереңдік артқан сайын массивтің шекті күйге өтуіне қарсылығының сандық тұрғыдан ұлғаюын көрсетеді. H/Lкр көрсеткішіне байланысты жер бетінің шөгу мульдасы өлшемінің экспоненциалдық тәуелділігі белгіленді, бұл әлеуетті опырылу аймағының ауданын болжауға мүмкіндік береді. Қажетті пасталы закладка массиві беріктігінің осы қатынасқа экспоненциалдық тәуелділігі анықталып, шекаралық күй жағдайында оның ең төменгі көтергіш қабілетіне қойылатын талаптарды айқындайды. Алғаш рет жойылмаған жерасты қуыстарының айналасындағы тау жыныстары массивінің шекаралық күйінің жер бетінің үдемелі опырылуына дейінгі дербес геомеханикалық күй ретінде бар екені сандық тұрғыда негізделді. Алынған тәуелділіктер жойылмаған қуыстар үстіндегі массивтің геомеханикалық жай-күйін болжауға, олардың критикалық геометриялық параметрлерінің ауқымын анықтауға және шекаралық күйдегі қуыстарды уақтылы айқындауға мүмкіндік береді. Әзірленген тәсіл жер бетінің кенет опырылуын болдырмау мақсатында жер бетінен цементтелген пасталы закладка параметрлерін инженерлік негіздеуде қолданылуы мүмкін.

Негізгі сөздер: *сандық модельдеу, жерасты қуысы, қабатты әртекті тау жыныстары массиві, Хоек–Браун критерийі, шекаралық күй, құлау, пасталы закладка.*

Численное моделирование критических условий формирования предельного состояния массива горных пород вокруг подземных пустот железорудных месторождений

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Аннотация. Целью работы является разработка подхода к количественной оценке и прогнозированию состояния массива горных пород вокруг непогашенных подземных пустот в предельном состоянии, опасном к потере устойчивости и обрушения земной поверхности, на основе численного моделирования. Исследование выполнено с использованием численного моделирования напряженно-деформированного состояния (НДС) слоистого скального массива методом конечных элементов в программном комплексе RS2. Для адекватного воспроизведения механического поведения трещиноватых пород Криворожского железорудного бассейна учтены стратиграфические особенности массива. Применен нелинейный критерий прочности Хоека-Брауна с учетом индекса геологической прочности (GSI). Установлено существование приграничного (приближенного к разрушению) геомеханического состояния массива горных пород вокруг непогашенных подземных пустот, формирующегося между устойчивым и неустойчивым и характеризующегося наличием деформационной связи с земной поверхностью. Выявлена устойчивая логарифмическая зависимость нижней и верхней границ критического соотношения $H/L_{кр}$ от глубины залегания пустоты, количественно отражающая возрастание сопротивления массива переходу в предельное состояние с увеличением глубины. Установлена экспоненциальная зависимость размера мульды сдвижения земной поверхности от показателя $H/L_{кр}$, что позволяет прогнозировать площадь потенциальной зоны обрушения. Выявлена экспоненциальная зависимость требуемой прочности пастового закладочного массива от соотношения, определяющая минимальные требования к его несущей способности в условиях приграничного состояния массива. Впервые количественно обосновано существование приграничного состояния массива горных пород вокруг непогашенных подземных пустот как самостоятельного геомеханического состояния, предшествующего прогрессирующему обрушению земной поверхности. Полученные зависимости позволяют осуществлять прогноз геомеханического состояния массива над непогашенными пустотами, определять диапазон их критических геометрических параметров и своевременно идентифицировать пустоты, находящиеся в приграничном состоянии. Разработанный подход может быть использован для инженерного обоснования параметров цементированной пастовой закладки с поверхности с целью предупреждения внезапных обрушений земной поверхности.

Ключевые слова: *численное моделирование; подземная пустота; слоистый неоднородный массив; критерий Хоека-Брауна; приграничное состояние; обрушение; пастовая закладка.*

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